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Synthesis of high-linearity array structures

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Abstract

The problem of synthesis of array structures capable of providing a highly linear voltage response has been studied in detail. Two possible approaches have been suggested. The first one supposes synthesis of the series array by means of interferometer cells with sinusoidal responses. The second approach supposes synthesis of highly linear differential array structures by means of interferometer cells with $|\sin(x)|$-like voltage responses at bias current $I_b = I_c$. When the interferometer cells are replaced by parallel arrays, a high-performance parallel–series differential structure is formed. The traveling wave amplifier design is suggested to avoid limitations resulting from the distributed nature of long series arrays.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Recently SQIF (superconducting quantum interferometer filter) structures, non-periodic arrays of dc Josephson junction interferometers, were suggested [1–4]. The SQIF voltage response shows a single delta-like peak at zero magnetic field. In the case of vanishing inductances, the voltage response can be expressed analytically for both parallel and serial SQIFs. The parallel SQIF response to a homogeneous magnetic field B is

$$V_{\text{par}}(B) = V_c\sqrt{(I_b/I_c)^2 - |S_K(B)|^2}, \tag{1}$$

where

$$S_K(B) = \frac{1}{K}\sum_{k=1}^{K}\exp\left(\mathrm{i}\frac{2\pi}{\Phi_0}B\sum_{m=1}^{k-1}s_m\right),$$

I_b—bias current, K—number of Josephson junctions, I_c—total critical current, s_m—area of the mth interferometer cell.

The serial SQIF response to a homogeneous magnetic field is a sum of the interferometer cell responses:

$$V_{\text{ser}}(B) = V_c\sum_{n=1}^{N}\sqrt{(I_b/I_c)^2 - |\cos(\pi B s_n/\Phi_0)|^2}, \tag{2}$$

where N is the number of dc interferometers. Different areas s_k mean different magnetic fluxes coupled into the cells and, therefore, different periods of the cell responses.

Our analysis of both formulae (1) and (2) shows that it is impossible to provide a highly linear response by any set of the cell areas. In order to provide a highly linear response, one should also change the amplitudes of the cell responses.

The purpose of the work is to develop an approach for synthesis of the serial arrays and series–parallel structures of dc interferometers capable of providing very linear triangular voltage response.

2. Synthesis of high-linearity series array structures

2.1. Periodic voltage response

If the voltage response of a serial array of dc interferometers to a homogeneous magnetic field B is a periodic even function, we can develop the response in Fourier series as follows:

$$V(B) = \sum_k a_k\cos(k\omega_0 B). \tag{3}$$

One can consider array cells providing sinusoidal voltage responses with frequencies proportional to the cell areas; basic frequency ω_0 can be expressed by basic area s_0 in the following way:

$$\omega_0 = (2\pi/\Phi_0)\cdot s_0. \tag{4}$$

Let us now consider a periodic triangular response. In this case, harmonic amplitudes should show a sinc-like behavior:

$$a(k\omega_0) = A\sum_k\frac{\sin^2(k\omega_0\Delta B/2)}{(k\omega_0\Delta B/2)^2}, \tag{5}$$

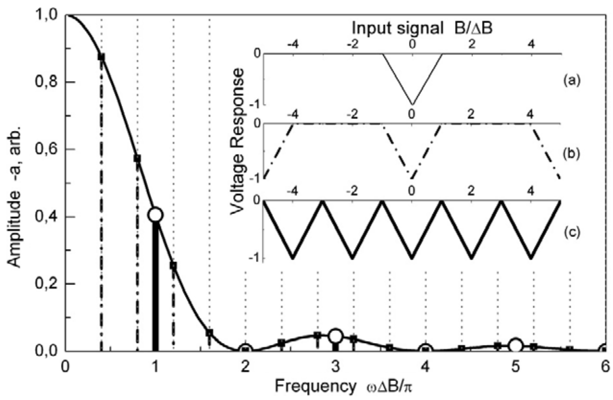

Figure 1. Spectrum of the triangular voltage response to magnetic field. Solid line—continuous spectrum of the voltage response with a single triangular peak ((a), inset). Filled and open circles with dropped lines—spectra of the periodic triangular voltage responses (b) and (c). The inset shows the responses.

where $2\Delta B$ is the width of the triangular pulse, $\omega_0 = 2\pi/B_T$ and B_T the period of the voltage response to magnetic field B. Figure 1 presents spectra of the responses with the same triangular pulse width ΔB and different periods $B_T = 5\Delta B$ (dash–dotted lines) and $B_T = 2\Delta B$ (thick solid lines). The inset shows these periodic responses ((b) and (c)), as well as a response with a single triangular peak which corresponds to a continuous spectrum shown by a solid line (a). It is significant that the spectrum of the voltage response with minimum period $B_T = 2\Delta B$ contains only odd harmonics with amplitudes decreasing monotonically with harmonic number k as $1/k^2$.

When a dc interferometer with critical current I_c is biased well above critical current ($I_b > 1.5I_c$), its voltage response to magnetic field becomes close enough to a sine wave. These interferometer cells may be used to compose a serial array providing a periodic linear voltage response. The array ought to consist of many groups of identical cells, i.e. with equal areas. Each group provides corresponding spectral component. Amplitudes of these spectral components are achieved by varying the number of cells in the groups. This approach to spectral line synthesis using groups of interferometer cells with equal areas is illustrated in figure 2.

Figure 3 shows the degree of linearity of triangular response versus cutoff frequency ω^* of the response spectrum. The cutoff frequency does not depend on the period B_T of the response, but the total number N^* of harmonic components needed does depend on B_T. In case of a response with minimum period $B_T = 2\Delta B$, the total number N^* is half the normalized cutoff frequency $\omega^*\Delta B/\pi$.

Such an array providing a highly linear triangular response can be called an LRA—linear response array.

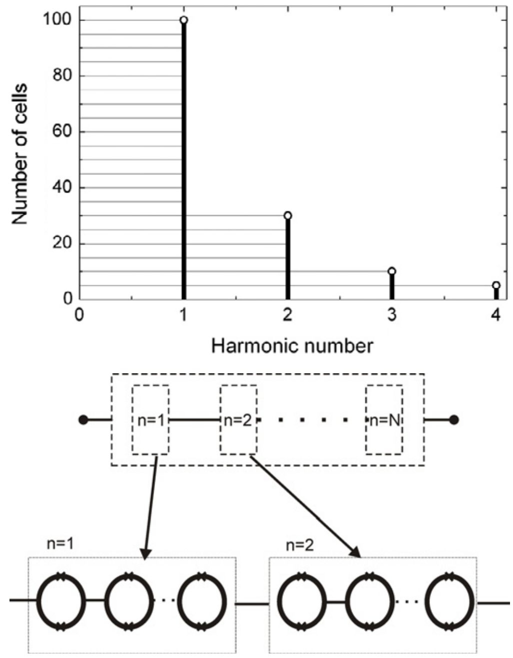

Figure 2. The spectral line synthesis by groups ($n = 1, 2, \ldots, N$) of identical cells (equal areas) with sinusoidal responses.

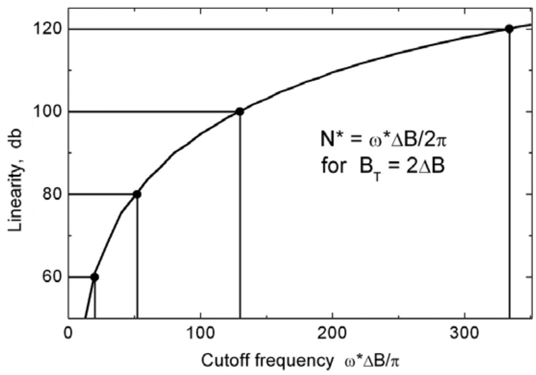

Figure 3. The triangular response linearity versus cutoff frequency ω^* of the response spectrum. In the case $B_T = 2\Delta B$, the total number N^* of harmonic components is half the normalized cutoff frequency $\omega^*\Delta B/\pi$.

2.2. Single triangular-peak response

We may compose a complete array using several LRAs connected in series. If the voltage responses of the LRAs are characterized by different periods B_T and the same width $2\Delta B$ of the triangular peak, the complete array will show a SQIF-like response with a single triangular peak. This array can be called an LR SQIF—linear response SQIF. The spectrum of the LR SQIF response consists of a set of basic frequencies $\omega_0\Delta B/\pi \leqslant 1$ and their harmonics in the upper range $\omega\Delta B/\pi > 1$, and amplitudes of the spectral components have a sinc-like frequency dependence. As for conventional SQIF, its response is characterized by a quasi-uniform discrete spectrum in the range specified by the maximum and the minimum cell areas.

2.3. Traveling wave amplifier

One should note that some limitations may result from the large area of an array with a large number of cells. Such an array becomes distributed. To overcome this problem, we suggested a traveling wave amplifier shown schematically in figure 4. The amplifier design includes two microwave lines coupled via interferometer cells of a serial array. The input wave

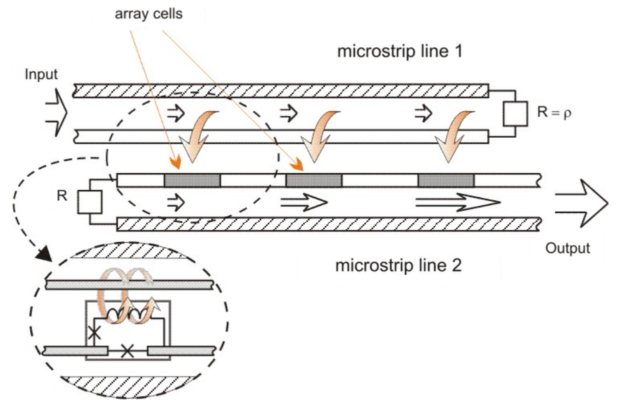

Figure 4. Traveling wave amplifier consisting of two coupled microwave lines. The inset shows coupling via an interferometer cell.

signal propagates along the first line and then is absorbed by a matched load. The propagating signal acts magnetically on the interferometer cells inserted in the second microwave line. Voltage responses of the cells induce the output wave running in the second line. Equal velocities of the input and output waves provide a sustained in-phase process and hence effective amplification of the output signal. This approach allows us to avoid size limitation in arrays and therefore achieve very high linearity. It also will lead to the design of wide band, highly linear amplifiers for the gigahertz frequency range.

3. High linearity differential parallel–series array structures

3.1. Two-cell differential circuit

In the limit of vanishing inductance, the dc interferometer voltage response (2) to a homogeneous magnetic field B at $I_b = I_c$ can be reduced as

$$V(B) = V_c|\sin(\pi B s_0/\Phi_0)|, \tag{6}$$

and the spectrum of the response shows a monotonic fall with harmonic number:

$$a(n\omega_0) = \frac{a_0}{n^2 - 1}. \tag{7}$$

If we remove all even harmonics from (7), we come just to spectrum (2) at $B_T = 2\Delta B$:

$$a(k\omega_0) = \frac{a_0}{(2k-1)^2}. \tag{8}$$

Some differences in the harmonic amplitudes occur only for initial harmonic numbers. It can be easily seen that applying half a flux quantum to the dc interferometer cell changes the sign of odd harmonic components. This fact leads to an idea of a differential scheme of unbiased and $\Phi_0/2$-biased interferometer cells to form the spectrum containing only odd harmonics. Figure 5(a) presents the unbiased voltage response $V(B)$ (a), the $\Phi_0/2$-biased response $[-V(B)]$ $(-b)$ and spectra of the responses. The inset shows the differential response of the unbiased and the $\Phi_0/2$-biased cells (sum of the shown a- and $-b$-responses).

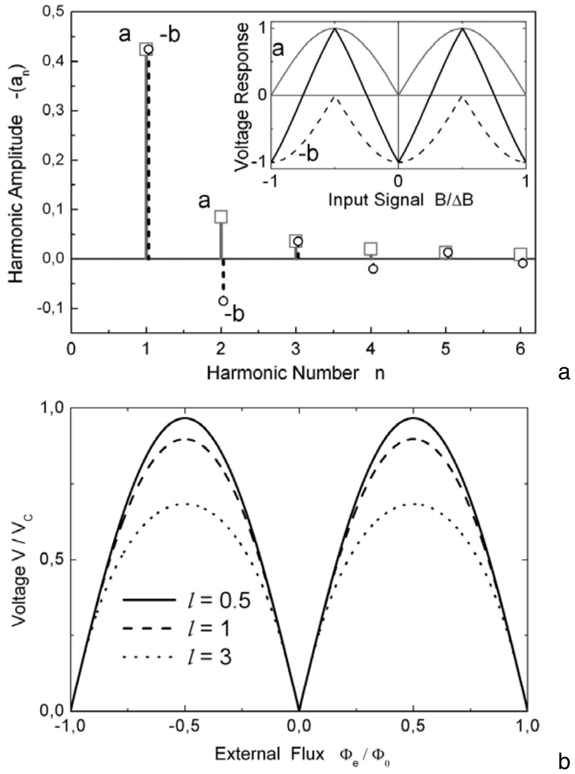

Figure 5. (a) The voltage response spectra of dc interferometers with vanishing inductances and current biasing $I_b = I_c$. Spectrum (a) relates to the interferometer which is unbiased magnetically, spectrum (b) to the inversed voltage response spectrum of the $\Phi_0/2$-biased interferometer. The inset shows the triangular voltage response on magnetic field for differential connection of the interferometers. (b) Voltage responses of dc interferometer with biasing $I_b = I_c$ at different normalized inductances l.

3.2. Inductance influence

Figure 5(b) shows the voltage response of a dc interferometer with biasing $I_b = I_c$ at different normalized inductances $l = \pi I_c L/\Phi_0$. We have found that the finite inductance l gives some additional decrease in amplitudes of only initial spectral components of the spectrum (5), while high harmonics remain the same and decrease as $1/n^2$. Therefore, we need to correct only these initial spectral components. As stated above, this can be done by means of additional cells with sinusoidal responses.

3.3. Differential array structure

The proposed differential circuit of two interferometers can be easy transformed into a differential array structure consisting of two serial arrays of dc interferometers with biasing $I_b = I_c$, where I_c is the critical current of the interferometers. In one array, each cell should be biased by $\Phi_0/2$. This differential array structure is shown schematically in figure 6(b). An additional flux $\Phi_0/4$ is applied to all interferometer cells to set the operating point in the middle of the array response leg.

One can increase the linearity of array voltage response. For this purpose, we should add to the array a few cells with sinusoidal responses. The cells are dc interferometer biased well above critical current ($I_b > 1.5I_c$). These additional cells

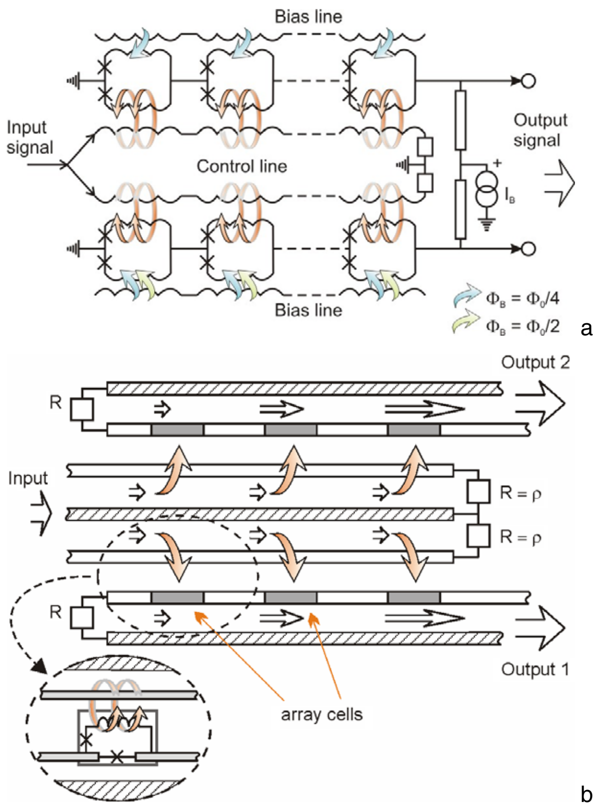

Figure 6. (a) Differential array structure consisting of two series arrays of dc interferometers with biasing $I_b = I_c$, where I_c is the critical current of the interferometers. In one array, each cell is biased by magnetic flux $\Phi_0/2$. Additional flux $\Phi_0/4$ is applied to all cells to set the operating point. (b) Differential traveling wave amplifier. The inset shows coupling via an interferometer cell.

are to correct the initial spectral components in (5) in order to approach the desired spectrum (8).

If the interferometer cells in both serial arrays are replaced by parallel arrays, we come to a high-performance parallel–series differential structure.

3.4. Differential traveling wave amplifier

To avoid limitations resulting from the distributed nature of LRA constituting a long serial array, we have proposed a traveling wave amplifier array structure (see figure 4). In case of the considered differential array structure, we come to a differential traveling wave amplifier shown schematically in figure 6.

The amplifier design includes the input microwave line coupled with two output microwave lines via interferometer cells of two serial arrays forming a differential structure. The input wave signal propagates along the input line and then is absorbed by a matched load. The propagating signal magnetically couples to the interferometer cells inserted in output microwave lines. The cell voltage responses form an output wave running along the output lines. Equal velocities of both the input and output waves provide an accurate in-phase process and hence the effective amplification of the signal. This approach allows using an arbitrary number of cells and thereby achievement of any desired linearity and voltage response amplitude.

4. Conclusion

The problem of synthesis of array structures capable of providing highly linear voltage response has been studied in detail and two possible approaches to the array building have been suggested. The traveling wave amplifier designs for the array structures are suggested to avoid limitations resulting from the distributed nature of long series arrays.

The developed array structures lead to the design of wide band, highly linear amplifiers for the gigahertz frequency range. The array-based amplifiers seem to be capable of providing essentially higher gain, linearity and dynamical range than the ones shown by the SQUID amplifier [5, 6]. In fact, the transfer factor $\mathrm{d}V/\mathrm{d}B$ is proportional to the number N of interferometer cells and the dynamic range increases as $\sqrt{N}$ for both parallel and series arrays. This results from the fact that in the case of the parallel array the output voltage noise decreases as $\sqrt{N}$ at fixed voltage response amplitude; in the case of the series array the output voltage noise increases as $\sqrt{N}$ and the voltage response increases as N.

In such a way, the use of a high enough number N of interferometer cells provides the required dynamic range, and the developed design approaches make the arrays LRAs with extremely high linearity of voltage response.

The experimental feasibility of an LRA looks quite realistic. For example, if we need the linearity level of 80 dB we have to provide $N^* = 25$ odd harmonic components (see figure 3) by means of 26 different groups with identical cells (see figure 2); numbers of the cells decrease as $1/(2k-1)^2$, $k = 1, 2, \ldots, 25$. The effective area of the cell in each group must be proportional to the corresponding spectrum component frequency $\omega_0(2k-1)$; the effective cell area which is responsible for the magnetic flux coming into the cell can be established by proper coupling between the cell and input signal line.

If we put 1000 cells in the first group, we can easy come to the total number of the cells in LRA $N = 1250$. The number seems quite optimistic; now we are going to test a series array of more than 4000 cells fabricated on the base of Hypres technology.

Acknowledgments

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